

Autumn Semester Examination 2018
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 409 (Integral Calculus)

Programme: B.Ed(S)

Level : IV

Writing Time: Three Hours

Full Marks: 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator *fx-82* or *fx-100* beside other writing materials.

Instructions : This paper contains FIVE questions. Answer any FOUR questions. Each questions carry 25 marks. Marks for each question or sub question are given in the brackets.

Question 1

- a. Evaluate $\int e^x \cos^3(e^x) \sin^3(2e^x) dx$. [6]
- b. Obtain a reduction formula for $\int x^m (\log x)^n dx$. Hence evaluate $\int x^4 (\log x)^3 dx$. [6]
- c. Prove that $\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx = \frac{\pi}{6\sqrt{3}} - \log \frac{2}{\sqrt{3}}$ [6]
- d. Find the area of the region bounded by the following curves $y = |x+1|+1$, $x = -3$, $x = 3$ and $y = 0$. [7]

Question 2

- a. Evaluate $\int \frac{e^{3x}}{e^{2x} + 7e^x + 10} dx$ [6]
- b. Solve $\int \frac{3 + 2 \sec x}{\tan x + 3 \sec x + 2} dx$ [7]
- c. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x \cos x} dx$ [6]
- d. Find the area of the region bounded by $\{(x, y) : x^2 \leq y \leq |x|\}$ [6]

Question 3

- a. Evaluate $\int \frac{x^3 - 2x^2 - 3x + 5}{x^3 - 2x^2 - 5x + 6} dx$ [6]
- b. Evaluate $\int (\sin 2x + \sin x) \sqrt{\cos^2 x + 4 \cos x + 3} dx$ [6]
- c. Solve $\int_{-2}^3 x \lceil x^2 \rceil dx$ [6]

- d. Find the volume obtained when area enclosed between the curves $2y^2 = x$ and $x^2 = 4y$ is revolved completely about the x-axis. [7]

Question 4

- a. Evaluate $\int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx$ [6]
- b. Solve $\int \frac{1}{2 - 3 \cos 2x} dx$ [6]
- c. Show that $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx = \frac{\pi}{2}(\pi - 2)$. [6]
- d. Calculate the volume of the solid when a region enclosed between the curve $y = 2x - x^2$ and line $y = x$ is revolved around the x-axis. [7]

Question 5

- a. Evaluate $\int \frac{a^{2x} + 2a^x}{\sqrt{a^{2x} + 5a^x + 6}} dx$ [6]
- b. Evaluate $\int \frac{dx}{\cos(x - a) \cos(x - b)}$ [6]
- c. Evaluate $\int_1^3 (x^2 + 5x) dx$ as limit of sums. [7]
- d. Find the area included between the curve $y = \cos x$ and the lines $x = 0$ and $x = 2\pi$. [6]